56. "A Simple Application of the Newtonian Law of Gravitation to Globular Star Clusters"

[*Einstein 1921f*]

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There is hardly any doubt that one can safely extrapolate Newton's law beyond the distances for which it has been verified. This confidence is also supported by the general theory of relativity, which provides a rational foundation for Newton's law such that an extrapolation to bodies that interact over larger distances appears all the more justified. However, the general theory of relativity allows for considerable deviations from the Newtonian law in a spatially finite universe, yet only in the case where the mean density of the stellar matter in the investigated, gravitating
[2] complex is not substantially larger than that of the rest of the universe.

In the following we shall apply the Newtonian law of gravitation to spherical star clusters. The main difficulty then is to find some consequences that can be tested, the reason being that our actual knowledge about the movement of stars in
[3] these clusters is exceedingly limited. The positional changes of stars during the time spans available to us are too small to be noticeable with our present means of observation. Furthermore, over these long distances, stars are not bright enough to allow for an investigation of their movements along the radius of vision by means of Doppler's principle. All one has is the image of the star cluster, projected parallel to the radius of vision, and even this only for the brightest stars in the cluster.

Yet one knows the approximate distances of the star clusters and with this their approximate true radius. This estimate is based upon the proven rule that stars of the same spectral type share approximately equal size and approximately equal absolute brightness. This assumption allows us to derive their distance from the apparent luminosity of stars, namely, by comparison with stars of the same spectral type, but closer to us. If one now knows the distance of a star that is close

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to us, then one can also determine the distance of the star cluster from us. From the apparent radius of the star cluster follows—at least within the order of magnitude—the true radius of the star cluster, and for the latter one has found values of 100–500 light years in this manner. [4]

[p. 51] It is probably safe to assume that the bright stars of the star cluster are approximately similar to the absolutely bright stars in our neighborhood. For the latter ones it has been established—using Doppler's principle—that they move relative to each other with a mean velocity of about 26 km/sec,[1] and we can probably assume that this is also the order of magnitude of the mean velocity of the bright stars of the star cluster relative to the center of gravity of the latter, the more so as it has been shown that the mean velocity of the stars of different spectral types also agree with each other *within the order of magnitude*.

We also assume that the distribution of stars in a stellar cluster is stationary insofar as the latter does not substantially change its radius and its star distribution (when considered from a statistical point of view) over a time during which individual stars of the cluster traverse a (curvilinear) path that is large compared to the radius of the cluster. It can hardly be questioned that this condition is satisfied for the radially symmetric and statistical distribution shared by many stellar clusters. Then it is possible to apply the virial theorem by Clausius to the star cluster as a whole, by treating individual stars as material points. In the case of Newtonian forces this yields, as H. Poincaré probably was the first to show, [5]

$$L = \frac{1}{2}\Phi . \qquad (1)$$ [6]

L is here the combined kinetic energy of all the stars of the cluster; Φ is the negative potential energy which is to be attributed to the cluster if the zero point of the potential energy of the stars is defined such that it vanishes when the distance between stars approaches infinity.

In order to be able to draw conclusions from equation (1), I make approximate assumptions about the structure of the cluster. I treat the stars of the cluster that are imaged on the photographic plate under short exposure as being all of equal mass m, and let N be the number of these types of stars within the whole cluster. Furthermore, I assume for the time being that the less luminous stars, that is, also the smaller ones in the cluster, do not substantially contribute to the gravitational field of the cluster, such that they can be neglected in the calculation of L and Φ. One then immediately gets, if v is the (quadratic) mean of velocity $\left(\mathrm{v} = \sqrt{\overline{\mathrm{v}^2}}\right)$,

[1]More precisely: relative to the center of gravity of the system to which they belong.

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$$L = N \cdot \frac{m\mathrm{v}^2}{2}. \quad (2)$$

For the calculation of Φ one must know the spatial density ρ for the stars of the cluster. It is well known that it can be represented in a satisfying manner by the empirical formula:

[7]

$$\rho = \frac{3}{4\pi}\frac{N}{a^3}\left(1+\frac{r^2}{a^2}\right)^{-\frac{5}{2}}. \quad (3)$$

Here, a is a length proportional to the radius of the cluster, $2a$ is the radius where the density has sunk to about 2% of the central density. Furthermore, ρm is the
[p. 52] mean density of the stellar matter within the cluster at a specific point. One does not commit a substantial error if one calculates Φ as if matter were distributed continuously with the density ρm. In this manner one gets

$$\Phi = A\frac{kN^2m^2}{a}. \quad (4)$$

[8] k is the gravitational constant, A is a numerical factor which I find to be about 0.6.

For the radius of the star cluster one gets from (1), taking (2) and (4) into account,

$$2a = 1.2\frac{kNm}{\mathrm{v}^2}. \quad (5)$$

If one puts for the star cluster in Hercules $N = 2000$, $m = 15$ masses of the sun
[9] $\mathrm{v} = 26\mathrm{km/sec}$, one obtains

$$2a = 0.65 \cdot 10^{18}\mathrm{cm} = 0.65\ \mathit{Lightyears}.$$

According to the apparent brightness of the brightest stars of the cluster, one has to assume the distance from us such that its radius cannot be less than 100 light years. Therefore, there must be an error in our assumption.

I had the opportunity to discuss the present difficulty in detail with my col-
[10] leagues at the institute of astrophysics in Potsdam. The result was that with our present knowledge of masses and distribution of fixed stars, *one* of my assumptions is substantially in error. The majority by far of the fixed stars in a star cluster had to be considerably lower in luminosity than the approximately 2,000 stars that appear on the photographic plate under short exposure, but without need to assume their masses as substantially smaller than those of the brightest stars. From pictures

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of the star cluster under long exposure and also from the distribution of fixed stars in our neighborhood, one can estimate that the number of fixed stars that contribute to the gravitational field of the cluster is about 100 times larger than we have assumed above. In this manner we obtain a radius of the star cluster of 65 light years, a value that is not so far off the lower limit that has been estimated in a different way.

The incompleteness of the material presently available from observations forces us for the time being to be content with this agreement in the order of magnitude. More precise results have to be based upon a better knowledge of star masses and star velocities. But *one* conclusion of considerable interest can be drawn from the agreement in the order of magnitude, and that is that the nonluminous masses contribute no higher order of magnitude to the total mass than the luminous masses.

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